A MODEL OF ECONOMIC AND EXERGETIC OPTIMISATION ANALYSIS A CASE STUDY OF SPACE HEATING SYSTEMS

UM MODELO ANALÍTICO DE OTIMIZAÇÃO ECONÔMICA E EXERGÉTICA UM ESTUDO DE CASO DE SISTEMAS DE AQUECIMENTO ESPACIAL

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ABSTRACT

Exergy represents the useful energy and it is interpreted as available work. Quality in energy conversion can be quantified through *exergetic* analysis based on the Second Principle of Thermodynamics (SPT). The design of energy systems is important in such way that the energy quality of an end-use is matched as much as possible to the energy supply, avoiding situations where a high quality supply is used for a low quality purpose. The extension of the efficiency criterion to include the SPT would result in fundamental changes in the way electric energy systems are designed and operated. The optimisation of a constraint space-heating system was investigated. This paper analyses the application of the SPT in the planning of electric energy systems case studies including exergetic efficiency achievement with economic considerations.

Keywords: Optimisation of Power Systems, Exergy, Integrated Resource Planning, Demand Side Management, Space Heating Systems.

RESUMO

Exergia representa a energia útil e isto é interpretado como trabalho disponível. Qualidade na conversão de pode ser quantificada através da analise exergética baseado no Segundo Princípio da Termodinâmica (SPT). O projeto de sistemas de energia é importante de tal modo que a qualidade de energia de um usuário final é combinada das formas possíveis de suprimento de energia, prevenindo o uso em situações onde o suprimento de energia de alta qualidade seja usado em objetivos de baixa qualidade. A inclusão do SPT no critério eficiência irá resultar em mudanças fundamentais na forma como são projetados e operados. A otimizarão de um sistema de aquecimento espacial restrito foi investigado. Este artigo analisa a aplicação do SPT na realização de estudo de caso de projeto de sistemas de energia elétrica incluindo eficiência exergética com considerações econômicas.

Palavras-chave: Otimização de sistemas de energia, Exergia, Planejamento Integrado dos Recursos, Gerenciamento pelo Lado Demanda, Sistemas de Aquecimento Espaciais.

1- INTRODUCTION

The understanding of the energetic and exergetic analysis and the corresponding efficiencies are presented in this paper, as well as means of performing economic optimisation. Exergetic analysis is a technique at the forefront of applied Thermodynamics research where any systems that utilise energy are assessed in the light of the Second Principle of Thermodynamics. All forms of energy transfer and transport can be represented by equivalent exergy transfers which are, in fact, the quantities of work that could be produced from the same types of energy transfer [1].

The First Principle of Thermodynamics (FPT) states that *energy can neither be created nor destroyed but can only be changed from one form to another* [2; 3; 4]. Thus, for any energy conversion process,

$$[\text{Input Energy}] = [\text{Useful Energy}] + [\text{Losses}] \tag{1}$$

where the energetic efficiency of a process,η based on the FPT, is defined as the ratio of the *Useful Energy* and the *Input Energy.*

The Second Principle of Thermodynamics (SPT) states that heat cannot be directly converted to work without any other effect. For a heat engine, low temperature waste heat cannot be avoided [3; 4; 5]. In other words SPT states that during any energy transformation, *the quality of the energy, as measured by its ability to perform work (exergy) degrades or at most keeps its original state,*

$$[\text{Exergy(Input Energy)}] \geq [\text{Exergy(Useful Energy)}] + [\text{Exergy(Losses)}] \tag{2}$$

where the energetic efficiency of a process, ε, based on the SPT is given by the ratio between the *Useful (Exergy)* and the *Input Exergy*.

It is evident from the above inequality that, unlike energy, exergy is not conserved in a process. The destruction of exergy is called *irreversibility*. Note the clear distinction between losses and the irreversible destruction of exergy. Losses are energy which is not useful to the particular conversion process and it is usually in the form of low temperature heat. Energy losses do not necessarily imply destruction, simply a conversion to another non useful form of energy[1]. On the other hand, the destruction of exergy in an energy conversion process normally implies a permanent decrease in the amount of available work.

In this study, the function exergy relating energy and exergy is expressed by,

$$Exergy = \alpha\ Energy \tag{3}$$

where α is a parameter laying between zero and one depending on other states such as temperature in the case of heat as well as on the available technology for converting that particular form of energy into work. Thus, for any energy conversion process.

$$\frac{\varepsilon}{\eta} = \frac{\alpha_2}{\alpha_1} \tag{4}$$

where α_1 and α_2 are the α's corresponding to the input and output sources of energy respectively. This important relation indicates that the First and Second Principle efficiencies and the energy/exergy conversion factors are not independent.

In the case of heat, η is limited by the Carnot efficiency of the ideal heat engine cycle, η_{Carnot}, and depends on the temperatures of the heat source, T_2, and of the cold sink, T_1, and the reference temperature $T_{o.}$ For practical reasons, the ideal Carnot efficiency cannot be achieved but can only be considered as an upper bound limit.

As examples of the above-mentioned ideas, the exergy content of electricity depends on the efficiency of the best electric motor available for the given kW rating (typically in the range $0.80 \leq \eta \leq 0.95$). Thus, for a 1 kW baseboard space heater, where the input is electricity and the output is heat at 20 °C, $\alpha_{electricity}$ is approximately 95% (based on the highest efficiency of electric motors) while α_{Carnot} is 6.8% (based on the ideal Carnot cycle efficiency with temperatures of 0°C and 20°C a baseboard 100% energetically efficient, its exergetic efficiency (ε) is 100*6.8/95 = 7.2%. Therefore, whereas 100% of the input energy is converted to a useful output in the form of heat, this process destroys 92.8% of the input exergy. This is a clear example of the possible wide discrepancies between energetic and exergetic efficiencies and of the significant irreversible loss of exergy even for a process which is 100% energetically efficient [1; 2; 5; 9].

Comparison between energetic and exergetic analysis guide us to the following reasoning: to perform energetic analysis of a process, it is necessary to treat it only as a black box with known input and output energies but knowledge of the process it is not required. On the other hand, to perform exergetic analysis it is necessary to know, not only the input and output energies, but also the details of the process as well as the technologies available to convert the input and output energy into work.

Heat related loads represent a significant fraction of the overall Quebec's consumption of electric energy in the Canadian economy. This article will focus in the Quebec's Canadian Province. The importance of this type of load, as well as the end-use energy associated with them, could be seen by the following [10; 11; 12; 13; 14; 15; 16]:

(i) The heat related loads represent more than half of the overall electric energy consumption of the province, around 58% for Quebec;

(ii) The space heating loads represents at least 77% of the heat related loads total consumption in the province;

(iii) The relative importance of electric heat related loads in general, as well as the space heating loads in particular, is greater in the residential sector than in the commercial and industrial sectors.

The relevance of optimisation studies resides in allowing the planner to better understand the system behaviour under different conditions. Although the solutions provided by the optimisation algorithms are most likely not to be found in the real world by reasons difficult to model, such as human preferences, optimal solutions outline the system behaviour under ideal conditions. Solutions provided by such optimisation studies are like landmarks, establishing limits for planning purposes under different objective functions.

The importance of optimisation is its ability to systematically find feasible designs among the infinite choices that satisfy a given set of equality and inequality constraints and simultaneously, minimise a predetermined objective function. A general objective function can be defined which assigns different weights to each natural resource, energy conversion device regarding to their energy, exergy content and the costs associated with them. Such formulations permit the planner to assign different values to each individual resource, including the possibility of differentiating between energy and exergy. In many situations the minimisation of one of the variables does not necessarily correspond to the minimisation of another. Thus, a compromise must be made by the planner to ensure that an acceptable balance among all variables can be reached. As an example, an energy policy may assign a higher value to oil, relatively to other resources, because of its scarcity, greater environmental impact and its dependence on foreign imports.

The present paper applies the optimisation principles, described above, to a realistic space-heating problem. This problem is first analysed from the FPT and SPT by minimising the total energetic and exergetic use at the natural resources level. The minimum energy and exergy solutions are then compared with the minimum cost designs. Finally, different types of cost incentives are

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[1] NOTE: There are numerous examples of energy losses with useful applications such as cogeneration [6; 7] and space heating "heat gains" due to cross-effects [8].

studied with the intent of forcing the minimum cost and minimum exergy solutions to coincide.

2 - SPACE HEATING MODEL

The application of optimisation techniques to the planning of heat related end-use (particularly space-heating) is important in exergetic analysis for the following reasons:

(i) There exist many space heating system alternatives because of the wide spectrum of possible energy sources and energy conversion devices and they should be assessed;

(ii) Heat related loads usually have a wide difference between the FPT and SPT efficiencies;

(iii) To make an intelligent choice among the many alternatives, a systematic approach is required. Such a choice may favour a design alternative which does not necessarily have the highest FPT efficiency but which has a high SPT efficiency.

(iv) Similarly, the minimum cost design with the existing electricity tariffs and fuel prices should be compared with other designs which maximise First and Second Principle efficiencies.

Figure 1 presents a model with only one end-use, namely the overall space heating requirements, (Q) and two natural resources, hydro resources (H, e_9) and fossil fuel (F, $e_7 + e_8$). The fossil fuel resources feed both the direct heating energy conversion (DH, e_7) equipment and the thermoelectric power plants (PP_th, e_8), while the hydro resources (H) are used for electric power generation through the hydro power plants (PP_hy). The amount of electricity produced, e_6, feeds the transmission system, TL. The transmission system feeds electrical energy, e_5 to the electrical energy conversion devices. In this model, four space heating alternatives were considered:

(i) Direct fossil fuel (DH), e_1;
(ii) Electric baseboard (BB), e_2;
(iii) Electric air-to-air heat-pump (HP_aa), e_3;
(iv) Electric ground-to-air heat-pump (HP_ga), e_4.

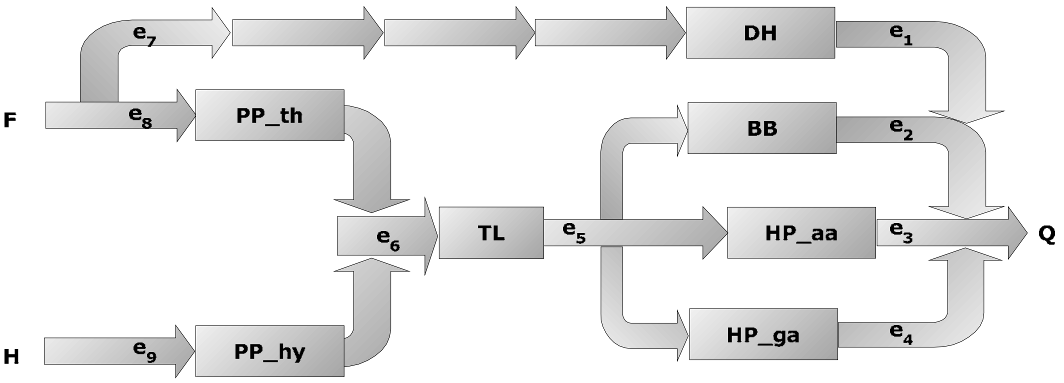

Figure 1 – Space heating model.

To perform the optimisation of the space heating model shown in Fig. 1 it is necessary to build the set of equations that characterise this problem, that is,

$$e_1 + e_2 + e_3 + e_4 = Q \tag{5}$$

$$\frac{e_1}{\eta_{DH}} = e_8 \tag{6}$$

$$e_6\, \eta_{PP\text{-}th} + e_7\, \eta_{PPhy} = \frac{e_5}{\eta_{Tl}} \tag{7}$$

$$\frac{e_2}{\eta_{BB}} + \frac{e_3}{\eta_{HP_{aa}}} + \frac{e_4}{\eta_{HP_{ga}}} = e_5 \tag{8}$$

The equivalent **A** matrix and **b** vector are given by,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{\eta_{DH}} & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & \frac{1}{\eta_{BB}} & \frac{1}{\eta_{HP\text{-}aa}} & \frac{1}{\eta_{HP\text{-}ga}} & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\eta_{Tl}} & \eta_{PP\text{-}th} & \eta_{PP\text{-}hy} & 0 \end{bmatrix} \tag{9}$$

$$\mathbf{b} = [\,Q \quad 0 \quad 0 \quad 0\,]' \tag{10}$$

where the symbol “ ´ ”, in equation 10, denotes the transpose of the vector.

3 - ECONOMIC ANALYSIS OF SPACE HEATING

The necessary input economic and other relevant data for the analysis of space heating options and the following comments apply to each row of the table (see Table 1):

(1) The capital cost for the alternative electric baseboard is the cheapest. The alternative ground-to-air heat pump has the highest capital cost and it represents more than 34 times the corresponding cost of the baseboard alternative. It must also be noted that the capital cost figures shown for the heat-pumps correspond to 75% of the actual cost of the device as the remaining 25% is associated with the cooling mode of the heat-pump;

(2) The life expectancy of electric baseboards is assumed to be the longest since the other options have more moving parts and are more likely to fail;

(3) The opportunity cost rate is the interest rate, on a yearly basis that would be earned above inflation if the capital

spent had been invested in the market;

(4) The efficiency or coefficient-of-performance varies significantly, ranging from 81% (for direct space heating) to 300% (for heat-pump ground-to-air). Note that the electric baseboard option is assumed to have an efficiency as measured by the First Principle of 100%, since it is assumed that all electric energy is converted to low temperature heat;

(5)The price of fuel to the customers is around 177% of the fuel price to the thermal power-plants. In this study, it is assumed to be the same for all three regions considered;

(6) The energy escalation rate per year is the increase above inflation of the fuel costs and electricity tariffs;

(7) Direct space heating and the ground-to-air heat-pump alternative have the highest maintenance rates (percentage of the initial capital cost per year);

(8) The number of hours of operation was assumed to be constant at 3.000 hours per year for all space heating alternatives considered; and

(9) Finally, the electric rates were considered for Quebec.

Before optimising the various design alternatives, it is useful to examine the life costs of the space-heating options at the customer level. The term life cost represents the cost to the customer for the expected life of the space heating device including, capital, maintenance and operational costs. The capital cost includes the initial investment and the opportunity cost. The operational costs involve both the maintenance and the energy costs (based on fuel rates or electric tariffs).

The average life costs for several alternatives considered for the space heating system (see Table 2 and Figure 1). Note that, strictly from the economic point of view, the *direct space-heating* alternative is the most economic one for the customer in terms of life costs for Quebec (see Table 2). The alternative *heat-pump ground to air* is the most expensive for the region studied. Finally, the life costs of the heat pumps are 2 to 32% cheaper in New York and Ontario

The average life cost separated into energy, maintenance and capital costs of the different space heating alternatives in Quebec (see Tables 3).

Table 1 – Relevant economic data for different space heating alternatives

Space Heating Alternative	**Electric Baseboard**	**Heat-pump air-to air**	**Heat-pump ground-to-air**	**Direct space heating**
1. Capital cost ($ / kW of end-use)†	49	857	1.713	117
2. Life expectancy (years)	20	15	15	15
3. Opportunity cost rate (% / year.)	4.0			
4. Efficiency (%)	100	170 ‡	300 ‡	81
5. Oil price	4.8 $/GJ (17.4 $/MWh) (for thermal power-plants)		8.5 $/GJ (30.7 $/MWh) (for customers)	
6. Energy escalation rate (% / year)	2.0			
7. Maintenance (% of capital cost / year)	1.0	1.5	2.0	2.0
8. Operation (hours / year)	3.000			
9. Electric rate,(¢ / kWh)*	6.52			

Note: † Quebec's Canadian $;‡ Coefficient of performance;
* Source: [13]

Table 2 – Average life cost to customers for different space heating alternatives

Space Heating Alternative	**Direct space heating**	**Electric Baseboard**	**Heat-pump air-to-air**	**Heat-pump ground-to-air**
1. Average life costs ‡ - (¢ /kWh of end-use)	5.00	8.27	8.37	10.55
2. Average life costs (%) - (Direct space heating =100)	100	165	167	211

† Source: [13]‡Canadian $

Note that (see Tables 3):

Table 3 – Energy, maintenance and capital costs at the customer level for different space heating options for Quebec

Space Heating Alternative		**Direct Space Heating**	**Electric Baseboard**	**Heat-pump air-to-air**	**Heat-pump ground-to-air**
1. Energy costs	¢/kWh	4.45	8.07	4.51	2.55
	(%)	(89.0)	(97.6)	(53.9)	(24.2)
2. Maintenance costs	¢/kWh	0.08	0.02	0.43	1.14
	(%)	(1.6)	(0.2)	(5.1)	(10.8)
3. Capital costs	¢/kWh	0.47	0.18	3.43	6.86
	(%)	(9.4)	(2.2)	(41.0)	(65.0)
4. Total	¢/kWh	5.00	8.27	8.37	10.55
	(%)	(100.0)	(100.0)	(100.0)	(100.0)

(i) The energy cost for direct space heating represents 89% of the life costs. For the electric baseboard, the energy cost represents more than 97% of the life costs. For the heat-pump space heating alternatives the energy costs represent around 24.0 to 53.9 to 75% of life costs of the device;

(ii) The maintenance costs represent between less than 1% (electric baseboard) to around 10% (heat-pump ground-to-air in Quebec) of the life cost of the space heating alternative;

(i) The capital cost of the direct space-heating alternative is 9.4 % of the life-cost of this alternative. However for the heat-pump space-heating alternatives the capital costs represent always a significant proportion of their life cost, ranging from 41.0% to 65.0%. On the other hand, capital costs represent only a small fraction, less than 2.2%, of the life cost of the electric baseboard alternative.

4 - OPTIMIZATION WITH MIXED OBJECTIVES

The space heating system can be optimised from the points-of-view of cost, energy and exergy subject to the space heating model equations 5 to 8. From the above discussion, the minimum cost solution is clearly the option with 100% direct space heating. These three optimum solutions according to the optimisation criterion chosen assuming that there are no limit on the amount of space heat provided by each alternative (see Table 4). Thus, the "best design" heating alternative depends on whether cost or energy/exergy considerations are pre-eminent. For example, if the space heating alternatives electric baseboard and direct space heating are compared, then the first option has the highest FPT efficiency but the direct space heating has the highest SPT efficiency

Table 4 – Best space heating as a function of the minimisation criterion, unconstrained case

Minimisation Criterion	**Best Design (Unconstrained)**
1. Cost	100% direct heating
2. Energy	100% hydro power plants + 100% ground-to-air heat-pumps
3. Exergy	100% hydro power plants + 100% ground-to-air heat-pumps

In a more realistic case, where the outputs of the possible space heating conversion devices are limited, as is the case discussed later in this paper, the best designs may involve combinations of all heating devices. In such cases, all designs where only energy and exergy are considered in the objective function (zero weight for cost); the ground-to-air heat-pump is normally saturated because from both the First and Second Principles, this alternative is more efficient than all others. After saturating ground-to-air heat pumps, the other alternatives will be selected in accordance with the objective function and the constraints chosen. In such cases, the minimum energy and exergy designs are normally not necessarily equal.

The combination of energy and exergy together with costs as optimisation criterion gives further insight when comparing space-heating alternatives. The mixed optimisation criteria considered for the space heating model analysis are listed (see Table 5). Note that e_R and x_R represent respectively the amount of energy and exergy consumed at the *resource level* while c_D is the cost to the customer of the end-use energy conversion devices. The subscript “R” represents the fuel or electric rate in ¢/kWh charged to the customers (see Table 5). The explicit dependence of the cost c_D on r is described in equation 9. Note, as well, those optimisation criteria one and two (see Table 5) are equivalent to maximising the First and the SPT efficiencies. Criterion four (see Table 5) minimises with equal weight both the exergy, x_R, and the energy, e_R, consumed at the resource level. Optimisation criteria four, five, six and seven combine with different weights energy and exergy at the resource level together with the cost to the customers. The optimisation criterion eight takes into consideration the minimisation of the subsidised cost to the customers, $c_D(\theta)$, where θ is the parameter representing the given subsidy, of which three have been considered, fuel rates, initial investment and opportunity costs.

The space-heating model was again optimised for the criteria shown (see Table 5) but this time, considering the constraints. These were chosen as typical levels in a fossil-fuel-based utility:

(i) The maximum values of the air-to-air heat-pump, $e_{3,}$ and ground-to-air heat-pump, e_4, were 5% and 10% of the value of the overall specified space heating requirement, Q, respectively;

Table 5 – Optimisation criterion and objective function considered for the space heating model analysis

Optimisation Criteria	**Objective Function**
1. Energy	e_R
2. Exergy	x_R
3. Cost	c_D
4. Energy and exergy	$e_R + x_R$
5. Energy and cost77	$w_1 e_R + c_D$
6. Exergy and cost	$w_2 x_R + c_D$
7. Energy, exergy and cost	$(w_1 e_R + w_2 x_R) + c_D$
8. Cost with subsidies	$c_D(\theta)$

(ii) The hydro potential state, e_7, was limited to 30% of Q;
(iii) The remaining states were not constrained.

The simulated values of the energy and exergy states (see Tables 6 and 7) of the space-heating problem for some of the optimisation criteria listed (see Table 5). Note that (see Table 6 and 7),

(i) In contrast to the unconstrained case (see Table 4), in the constrained case the optimum solutions for a minimum x_R or minimum e_R or even minimum (x_R + e_R) are all distinct;

(ii) The upper bound limit of state e_4 (space heating from ground-to-air heat pump) was reached for all three optimisation criteria. This is due to the fact that the ground-to-air heat pump is more efficient from either the First or Second Principle perspectives;

(iii) The state e_2 (heating from electric baseboard) is different from zero only when the optimisation criterion is the minimisation of the energetic resources, e_R, at the natural resource level. In this case, the upper bound limit of the hydro-potential, e_7, is reached as well;

(iv) The use of thermal power plants is never part of the solution for all the criteria tested, that is, the state e_6 is always zero. If hydro-resources are not available, thermal power plants will be present to supply the heat-pump requirements of the optimum solution;

(v) The minimum cost solution requires only direct heating;

(vi) The first four exergy states have relatively low values, due to the fact that the exergy in low heat temperature sources is extremely low;

Table 6 – Optimum energy states for different optimisation criteria

Optimisation Criteria	**Min c_D**	**Min x_R**	**Min e_R**	**Min (e_R + x_R)**
States	End-use exergy (% of Q[†])			
x_1 (end-use direct heating)	2.7	2.6	1.8	2.3
x_2 (end-use electric baseboard)	0.0	0.0	0.5[‡]	0.0
x_3 (end-use heat-pump air-to-air)	0.0	0.0	0.3[‡]	0.3[‡]
x_4 (end-use heat-pump ground-air)	0.0	0.1[‡]	0.1[‡]	0.1[‡]
x_5 (electric load)	0.0	1.6	24.9	7.2
x_6 (fuel for thermal power-plant)	0.0	0.0	0.0	0.0
x_7 (hydro potential resources)	0.0	1.8	28.5[‡]	8.2
x_8 (fuel for direct space heating)	49.4	46.9	32.8	42.0
x_R (total exergy consumption at natural resource level)	49.4	48.7	61.3	50.2

[‡] Q = specified overall space-heating requirements.[†] Upper bound limit reached

(vii) The energy consumed at the resource level, e_R, is a maximum when minimising cost. The minimisation of the end-use cost, c_D, leads to the maximum energy consumption at the natural resource, e_R. However, the minimisation e_R leads to maximum exergy consumption. However the maximum exergy consumption occurs when minimising e_R. This behaviour is very interesting since it implies that the minimisation of the cost, energetic and exergetic resources are conflicting goals.

Table 7 – Cost and efficiencies of the space heating for different optimum criteria for Quebec

Optimisation Criterion	**Cost (¢/kWh of end use)**	**η (%)**	**ε (%)**	**Description of the states**
1. Min x_R or Min e_R or Min (x_R+e_R) (Unconstrained solution)	10.55	262.2	7.54	All the space heating from heat pump ground-to-air and all electric power supplied by hydro power plants.
2. Min c_D (Constrained solution)	5.00	81.0	5.53	All the space heating from direct oil or gas heating
3. Min x_R (Constrained solution)	5.28	83.9	5.60	Heat pump ground-to-air saturated power supplied by hydro power plants; the remaining space heating requirements supplied by direct fossil fuel space heating.
4. Min e_R (Constrained solution)	6.23	89.4	4.46	Heat-pump ground-to-air and air-to-air saturated to the limit, electric baseboard utilise up to the limit of the hydro power plants, the remaining space heating requirements supplied by direct fossil fuel space heating.
5. Min (e_R+x_R) (Constrained solution)	5.62	88.1	5.44	Heat pump ground-to-air and air-to-air saturated power supplied by hydro power plants; the remaining space heating requirements supplied by direct fossil fuel space heating.

The average life cost to the customer (see Table 7), the First and Second Principle efficiencies and the description of the states for five different optimisation criteria for the Quebec region. All the cost figures shown (see Table 7) are given in cents of Canadian dollars per kWh of end use (¢ / kWh). These costs were calculated over the life period of each individual device, considering the economic inputs shown (see Table 1) and equation 9. Several points should be stressed about the results shown (see Table 7). For each criterion the corresponding comments apply:

(1) For the unconstrained cases, shown in item one, all three criteria have the same solution, that is, all the space-heating requirements are met by ground-to-air heat pump. The cost for this case was 10.55 ¢/kWh, corresponding to the costs of the ground-to-air heat pump shown (see Table 2). Note, as well, that the variation in the cost is not in the same proportion as the electricity rates in the three regions studied, since the capital cost for a heat-pump is a major portion of its average life cost as emphasised (see Table 3). Both the First and Second Principle system efficiencies in this case are the highest among all the cases tested with values of 262.2 % and 7.54 %, respectively. Clearly the unconstrained cases are not normally realisable and are presented only for reference purposes. The importance to have such reference resides in allowing the planner to know the upper bound limits in the efficiencies, as well as, to compare different end-uses of energy from the points of view of the FPT and the SPT, as it will be shown later on;

(2) The minimum cost solution requires only direct space heating (item 2) and yields an average life cost of 5.00 cents per kWh. Since the oil cost for space heating purposes was considered to be constant for the regions studied and the minimum cost solution utilises only direct heating then the costs for the three regions for this optimum criterion are constant;

(3) Minimisation of the exergy consumption at the resource level is equivalent to maximisation of the Second Principle efficiency. Item three shows that $\varepsilon_{max,}$ for the constrained case, is achieved by saturation with ground-to-air heat pumps with their energy supplied by hydroelectric power generation. The remaining space heating requirements are supplied by direct space heating devices. Note that the cost for this alternative is not significantly different from the minimum cost solution. The cost is within 10% of the minimum cost solution. The reason for this (see Table 6) is that the minimum x_R and minimum c_D solutions differ only by the amount of ground-to-air heat-pump which is limited to only 5% of the space heating requirements. Later on, in this section, further discussion is given to design programs and incentives, which induce the maximum, ε and minimum cost solutions to coincide;

(4) Minimisation of the energy consumption at the resource level is equivalent to maximisation of the First Principle efficiency,η. Item four shows that $\eta_{max,}$ for the constrained case, is achieved by saturation with the ground-to-air and air-to-air heat pumps, some direct-heating as well as some electric baseboard heaters. The latter are increased until the hydroelectric resource limit is reached however no thermoelectric generation is required by the optimum solution. The costs, in this case, increase substantially when compared with the minimum cost solution, (from around 24%). The reason of this substantial increase is due to the increased use of air-to-air heat pumps and electric baseboards;

(5) Finally, the criterion combining exergy and energy consumption at the resource level (item 5) yields a mixture of the solution reached by items three and four. This optimum solution imposes that all heat pumps reach their limits, with the remaining space heating requirements being provided by direct fossil fuel heaters. Again, all the electric power is provided by hydropower generation. Note that the costs and the efficiencies have intermediate values compared with items three and four as would be expected;

(6) Comparing the minimum energy solution with the minimum cost solution at the customer level, note that, the variations in η are 8.4 percentage points representing a variation of 9.8% over the average value of the First Principle efficiency. The equivalent variation for η is 1.14 percentage points, representing a variation of 21.7 % over the average value of the Second Principle efficiency. The percent variation of the Second Principle efficiency is therefore much larger than the variation in the First Principle efficiency;

(7) Note that the cost difference between the minimum cost and the minimum exergy solution is less than half a cent per kWh, or around 6 % of the average cost for the four alternatives considered. Also the minimum energy solution yields, for all three regions studied, much higher costs (around 19% for Quebec) than the minimum exergy solution. This is an interesting fact indicating since, for this example that the maximisation of the resources from the First Principle not only is more costly but also provides the worst Second Principle efficiency.

Thus, the use of the FPT as the sole optimisation criterion does not lead to a rational use of resources from the perspective of the SPT which is viewed as a rational method of energy planning but results in the most expensive solution.

The minimisation of both energy and exergy consumption at the resource level provides an intermediate cost as well as a compromise between First and Second Principle efficiencies. It is important to analyse such intermediate solutions since extreme cases of maximum ε or minimum cost may not be achievable in practice.

The question that naturally follows is how to realise the above-mentioned strategies specially the one that maximises the natural resources as measured by the exergetic content. The following section discusses alternatives as to how to implement the maximisation of exergetic efficiency in the space-heating model.

The optimisation of the space-heating problem is now considered with weights in the energy and exergy consumption at the resources level combined with the overall cost at the customer level. This analysis is done to determine the relative weights that must be assigned to cost and to energy/exergy resources in order to achieve a desired solution (such as the minimum exergy solution). This weight then indicates to the planner how far the minimum cost solution is from the desired solution. Some results of such optimisation are presented (see Table 8).

Table 8 –Minimum energy/exergy weights (¢/kWh) in the linear programming objective to force the solution to be equal to the minimum s_R solution

Objective Function	**Desired Solution**	**Quebec**
1. Energy and cost (w e_R+c_D)	min x_R	na.
2. Exergy and cost (w x_R+c_D)	min x_R	42.2
3. Energy exergy and cost $w_1(e_R+x_R) + c_D$	min x_R	5.7

na. = not achievable

To interpret the results (see Table 8), consider, for example, in row 1, that, in New York, it is necessary to weigh the total energy resources, e_R, by 18.4 ¢/kWh so that the minimum solution for the objective function w e_R + c_D will be equal to the desired solution (in this case assumed to be the minimum x_R solution). This value of w is very high compared with the average cost of the minimum x_R which, (see Table 7), is 5.48 ¢/kWh. The results (see Table 8), imply that the minimum cost solution is relatively far from the desired minimum exergy solution and that to achieve the latter one must tax either the resources, e_R or x_R, or subsidise the cost, c.

5 - IMPLEMENTATION OF DESIRED OPTIMUM SOLUTIONS

Since maximising the First Principle efficiency alone is not sufficient, in general, to achieve the best use of natural resources, economic measures could be conceived that would make the most efficient solution according to the SPT the most economically attractive. Economic measures are strategies that serve to stimulate society to adopt technologies and energy use patterns compatible with a given philosophy. Another possible strategy is to increase the public consciousness about the importance of exergetic considerations in the planning of the utilisation of natural resources.

In this section, a number of economic measures are considered to induce users to conform to the requirements of the minimum exergy solution, assuming that users tend to consume energy in a manner consistent with minimum cost. The economic measures considered in this paper are:

(i) Subsidy in the initial capital cost investment, IC;

(ii) Subsidy in the opportunity cost rate, p, at the end-use device level;

(iii) Subsidy of the fuel and the electricity tariffs, r.

As noticed the difference between the minimum cost and maximum ε solutions in the end-use states (see Table 7) (e_i , i = 1 to 4) is the replacement of some direct heating by ground-to-air heat pumps. Therefore, the subsidies considered in this example are applied only to the ground-to-air heat pump so as to encourage customers to switch from direct heating. An alternative approach would have been to tax direct heating however this was not considered in this analysis. The minimum subsidy to conform to the minimum exergy is found by progressively increasing the subsidies until the minimum cost solution with subsidies is identical to the minimum exergy solution (note that the minimum exergy solution does not involve economic considerations).

The results (see Tables 9 and 10) of trials with the various proposed subsidies for the space-heating problem. The subsidies in the initial capital and opportune costs (see Table 9). Several points should be highlighted about the results (see Table 9):

(i) There are various possible combinations of initial capital and opportunities cost subsidies, which result in the same exergy solution. For each of them the total lifetime subsidy is 5.55 ¢/kWh;

(ii) The initial capital cost subsidy in the ground-to-air heat-pump in Quebec varies between 81% and 66% of the cost of the initial investment for an opportunity cost rate variation subsidy of 0 to 4% per year;

(iii) The initial cost subsidy over the lifetime of the ground-to-air heat-pump varies from 3.08 ¢/kWh to 2.50 ¢/kWh, as the subsidies in the opportunity cost rate increased from 0 to 4% per year. An increase in the opportunity cost rate from 0 to 4% per year corresponds to a lifetime subsidy of 0 to 3.05 ¢/kWh;

(iv) Note that the opportunity cost rate subsidy in % per year can be translated into ¢/kWh over the lifetime of the device. For example, comparing a subsidy in the opportunity cost rate of 2 with 4% per year implies in 1.73 and 3.05 ¢/kWh respectively. Note that this correspondence is non-linear.

Table 9 – Minimum % subsides for varying opportunity cost rates in the initial capital of the heat pump ground-to-air to induce the minimum exergy and minimum cost solutions to be identical in Quebec

Initial Capital subsidy		**Rate**		**Observations**
(%)	(¢/kWh)	Quebec rate subsidy %	(¢/kWh)	
50	3.43	28	4.72	
55	3.77	21	5.18	
60	4.12	13	5.64	
65	4.46	6	6.10	
70	4.80	0	6.56	No need for rate subsidy in Quebec (r_{QB} = 6.52 ¢/kWh).

Table 10 – Heat-pump ground-to-air initial capital subsidy and maximum rate for the minimum solution x_R be the minimum cost solution

Opportunity cost rate subsidy (% per year)	**0**	**1**	**2**	**3**	**4**
1. Initial Capital Cost Subsidy (%)	81.00	78.00	74.00	70.00	66.00
2. Initial cost average lifetime subsidy (¢/kWh)	3.08	2.97	2.84	2.68	2.50
3. Capital cost and opportunity cost (¢/kWh)	5.55	4.62	3.82	3.11	2.50
4. Opportunity cost-rate lifetime subsidy (¢/kWh)	0.00	0.93	1.73	2.44	3.05
5. Total average lifetime subsidy (¢/kWh)	5.55	5.55	5.55	5.55	5.55

6 - CONCLUSION

This paper has investigated the optimisation of a space heating system subject to constraints with four different alternatives: electric baseboard, ground-to-air heat pump, air-to-air heat-pump and direct fossil fuel heating. The design is based on the minimisation of the energetic or exergetic consumption at the natural resource level and the cost to the customer as well as combinations of these.

Since exergy is the ability of a given form of energy to be converted into any other form, it is argued in this paper that the most rational manner to optimise any energy system (not just space heating systems) is by minimising the exergy consumption at the natural resource level. In other words, that electric energy system should be designed by maximising their overall exergetic efficiency. The case here studied, showed that, although, in some special cases, the maximum energy efficiency solution may coincide with the maximum exergy efficiency solution, this is not true in general. Thus, in general, to design energy systems rationally, it becomes essential to explicitly include exergy in the objective function of the design problem.

In order to ensure that the system design corresponds to the minimum exergy solution, it is hypothesised here that the given energy end-uses will be met by those alternatives that minimise the cost to the customers. In other words, the end-users of energy will tend to choose the cheapest alternatives according to their lifetime costs. Of course, in a real system there is no guarantee that this hypothesis will be followed exactly because of reasons such as human preferences, convenience, lack of information or concern for the environment, but these have not been modeled in this paper.

Thus, different cost incentives were tested in order for the minimum exergy and minimum cost solutions to be identical. The costs incentives studied were subsidies in the initial capital cost investment, opportunity cost rate and the energy tariffs. Although the minimum exergy solution could be achieved with different combinations of subsidies, those involving only subsidies related to capital investments appear to be easier to implement.

It is noted that this paper dealt with the optimal design of an energy system with only one end-use, namely space heating, thus, the results serves only to illustrate the methodology. Nevertheless, the approach can be extended to more general systems with multiple end-uses.

REFERENCES

[1]. McGovern, J. A., "Exergy Analysis- a Different Perspective on Energy, Part 1: The Concept of Exergy", *Proceedings of the Institute of Mechanical Engineers*, Volume 24, pp. 253-262, 1990.

[2]. Klenke, W., "Useful Work, Exergy and Thermodynamic Potentials", *International Chemical Engineering*, Volume 31, No. 4, October, pp. 654-660, 1991.

[3]. Bejan, A., *Advanced Engineering Thermodynamics*, Wiley, New York, 1988.

[4]. Holman, J. P., *Thermodynamics*, 3rd. Ed., McGraw Hill New York, 1980.

[5]. Krenz, J. H., *Energy from Opulence to Sufficiency*, Praeger, 1980.

[6]. Perlman. M., Moore, G., "On-site Generation Through Waste Heat Recovery", *Canadian Electrical Association*, No. 8901-U-762, Montréal, Quebec, Canada, 1991.

[7]. Clark, E. L., "Cogeneration - Efficient Energy Source", *Annual Review Energy*, Volume 11, pp 275-294, 1986.

[8]. Moreau, A., Stricker S., "Assessment of the Impact of Internal Heat Gains on The Thermal Loads in the Residential Sector", *Electricity 1994*, (Toronto: Canadian Electrical Association) p. 19, 1994.

[9]. Oliveira F., D., Galiana F. D."A Model for the Planning of Electric Energy Systems. Including Energetic Considerations", *The Institute of Electrical and Electronic Engineers Power Industry Computer Applications Conference*, May, Utah, pp. 539-546, 1995.

[10].Gelbard, E., Li, C., "1992 Residential Sector End-Use Forecast", *Main Report*, Ontario Hydro, Toronto, 1993.

[11]. Zhu, J. L., Lodola J. C., "1992 Commercial Sector End-Use Forecast", *Main Report*, Ontario Hydro, Toronto, Canada, 1993.

[12].Low, S.,, "1992 Industrial Sector End-Use Forecast", *Main Report*, Ontario Hydro, Toronto, 1993.

[13].Canada, Statistics,, *Quarterly Report on Energy Supply-Demand in Canada*, 1994.

[14].Quebec Government,, "Energy in Quebec 1992", *Ministère de l'Énergie et des Resources*, Canada, 1992.

[15].Quebec Government, , "Forecast Electricity Demand in Quebec, Development Plan for 1993", Volume 6, Montréal, Quebec, Canada, 1992.

Oliveira F., D., , "Electric Energy System Planning and the Second Principle of Thermodynamics", PhD Thesis, McGill University, 199p., Montreal, Quebec, Canada, 1995